Transition to A-Level Maths Workbook

Algebraic Manipulation and Proof

Q1. Expand and simplify $(2x + 5y)(3x - 8y)$

(Total 3 marks)

Q2. Expand and simplify fully $(x - 3)(x + 2)(x + 5)$

(Total 3 marks)

Q3. Rearrange $y = \frac{4 - 3x}{x - 5}$ to make x the subject.

(Total 4 marks)

Q4. Circle the identity.

$(x - 3)^2$ $(x - 3)^2 > 5$

$(x - 3)^2 = 1 - 6x$ $(x - 3)^2 \equiv x^2 - 6x + 9$

(Total 1 mark)

Q5. Work out the values of a and b in the identity

$5(7x + 8) + 3(2x + b) \equiv ax + 13$

(Total 4 marks)

Q6. Prove that $3(x + 1)(x + 7) - (2x + 5)^2$ is never positive.

(Total 5 marks)

Q7. Expressions for consecutive triangular numbers are

$\frac{n(n+1)}{2}$ and $\frac{(n+1)(n+2)}{2}$

Prove that the sum of two consecutive triangular numbers is always a square number.

(Total 4 marks)

Q8. Two integers have a difference of 3.
The difference between the squares of the two integers is three times the sum of the integers.

For example, $13 - 10 = 3$, $13^2 - 10^2 = 169 - 100 = 69$

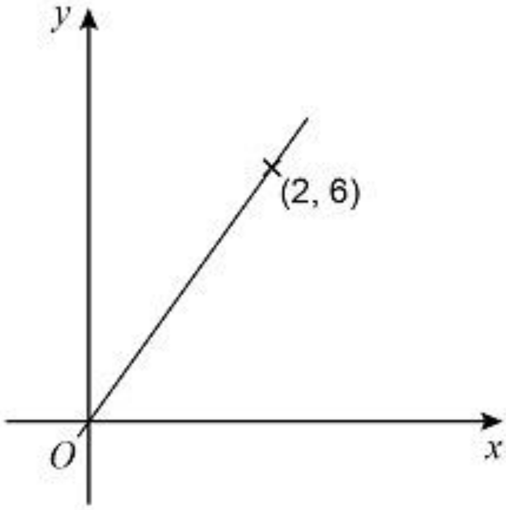
And $3 \times (13 + 10) = 3 \times 23 = 69$

Prove this result algebraically.

(Total 4 marks)

Straight Lines and Circles

Q1. A straight line passes through O and (2, 6)



Circle the equation of the line.

$y = x + 4$ $\quad\quad$ $y = 6$ $\quad\quad$ $y = 3x$ $\quad\quad$ $y = \frac{1}{3}x$

(Total 1 mark)

Q2. The equation of a straight line is $\quad 2y = 3x + 5$

Circle the gradient of the line.

$\frac{2}{3}$ $\quad\quad$ $\frac{3}{2}$ $\quad\quad$ 3 $\quad\quad$ 5

(Total 1 mark)

Q3. (a) Show that the lines $y = 3x + 7$ and $2y - 6x = 8$ are parallel.

Do **not** use a graphical method.

(3)

(b) Is the point (−5, −6) above, below or on the line $y = 3x + 7$?

Tick **one** box.

☐ Above ☐ Below ☐ On the line

You **must** show your working

Do **not** use a graphical method.

(2)

(Total 5 marks)

Q4. Line A has equation $y = 4x - 1$

Line B is

perpendicular to line A

and

passes through the point (8, 5)

Work out the coordinates of the point where line B intersects the x-axis.

(Total 4 marks)

Q5. A circle has equation $x^2 + y^2 = 4$

Circle the length of its radius.

2 4 8 16

(Total 1 mark)

Q6. The equation of a circle is $x^2 + y^2 = 9$

Work out the length of the diameter.

Circle your answer.

3 6 9 18

(Total 1 mark)

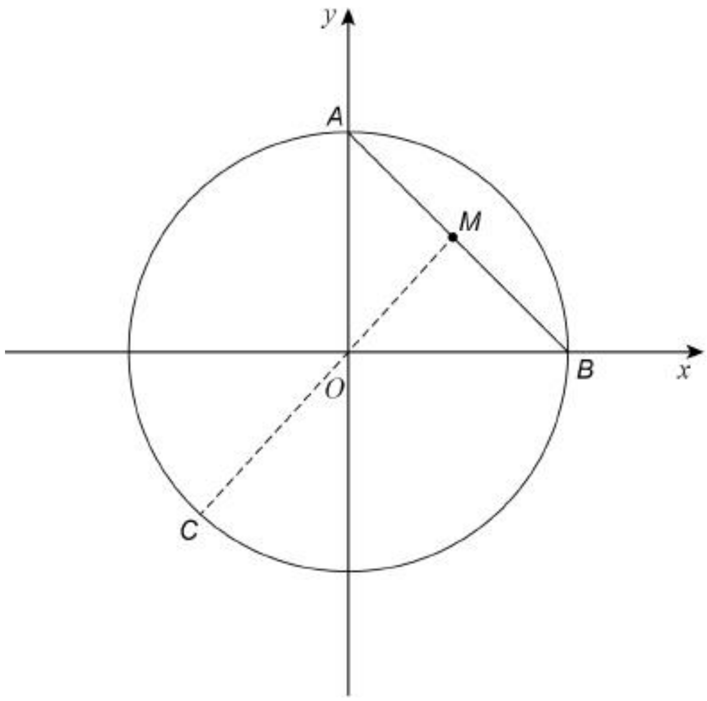
Q7. *A*, *B* and *C* are points on the circle $x^2 + y^2 = 36$ as shown.

A is on the y-axis.

B is on the x-axis.

M is the midpoint of *AB*.

COM is a straight line.



(a) Show that the coordinates of *A* are (0, 6)

(1)

(b) Work out the coordinates of *B*.

(1)

(c) Show that the equation of the straight line passing through *C*, O and *M* is $y = x$

(2)

(d) Work out the coordinates of *C*.

Give your answers in surd form.

(3)

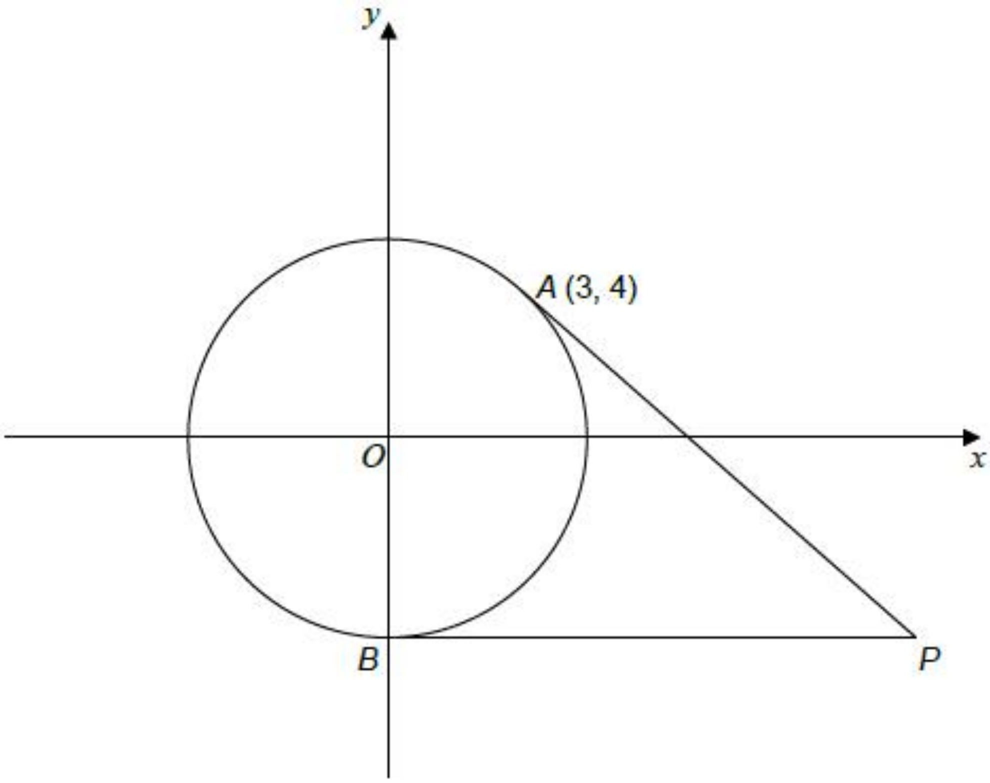
(Total 7 marks)

Q8. *A* and *B* are points on the circle with equation $x^2 + y^2 = 25$

A is (3, 4)

B is a point on the y-axis.

PA and *PB* are tangents.



(a) Show that the coordinates of *B* are (0, −5)

(1)

(b) Give a reason why *PA* = *PB*

(1)

(c) *P* is the point (a, b)

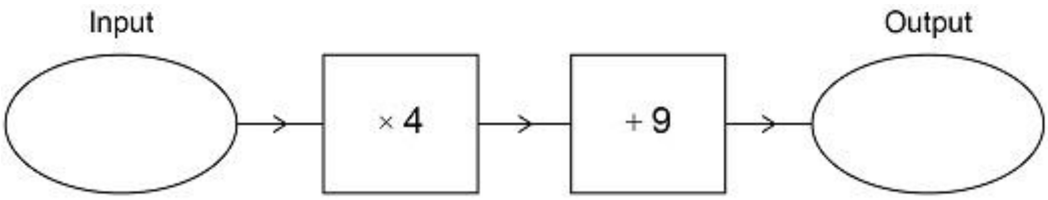
Work out the values of a and b.

(4)

(Total 6 marks)

Functions

Q1. Here is a number machine.



Work out the output when the input is 16

(Total 1 mark)

Q2. $f(x) = x^2 - x^3$

Circle the value of $f(-3)$

18   −18   36   −36

(Total 1 mark)

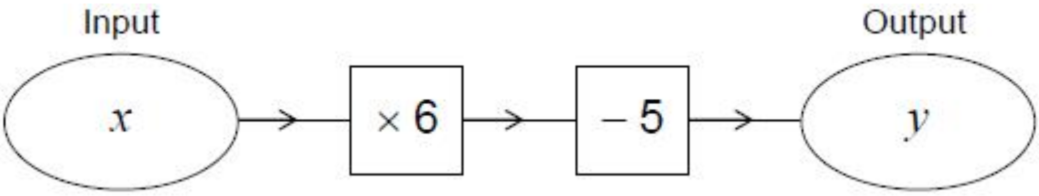
Q3. $f(x) = 3x$ and $g(x) = x^2$

Circle the expression for $fg(x)$

$3x^2$ $9x^2$ $3x^3$ $9x^4$

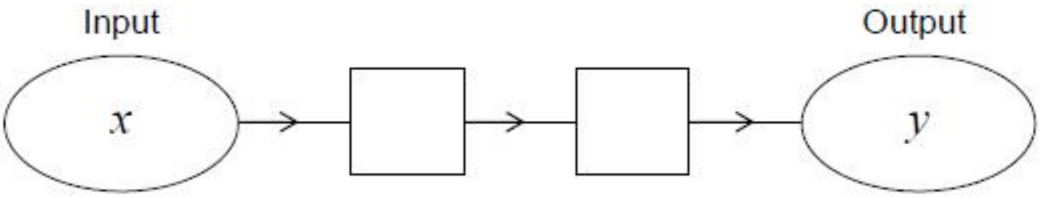
(Total 1 mark)

Q4. (a) Work out the output y when $x = 4$



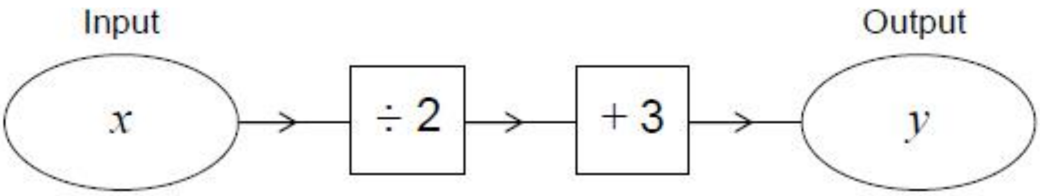
(1)

(b) Complete this number machine so that $y = 2(x + 7)$



(1)

(c) Here is a different number machine.



Which equation is correct for this machine?

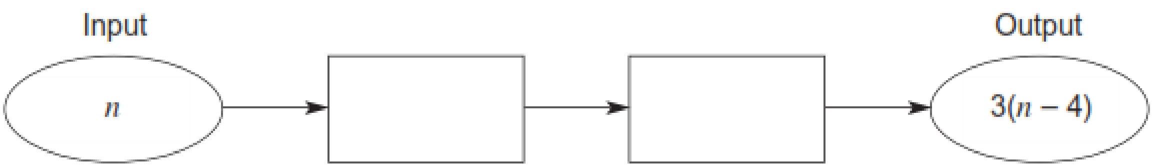
Circle your answer.

$y = \frac{x}{2} + 3$ $x = \frac{y}{2} + 3$ $x = \frac{y+3}{2}$ $y = \frac{x+3}{2}$

(1)

(Total 3 marks)

Q5. Here is a number machine.



(a) Write an operation in each box to make the number machine work.

(2)

(b) Work out the value of n when the input and output are equal.

(2)

(Total 4 marks)

Q6. For all values of x, $\mathrm{f}(x) = \frac{4x-3}{5}$

Work out $\mathrm{f}^{-1}(x)$

(Total 3 marks)

Q7. $\mathrm{f}(x) = 2x^2$

$\mathrm{g}(x) = x + 5$

Circle the composite function $\mathrm{fg}(x)$

$2x^2 + 5$ $\quad$ $2(x+5)^2$ $\quad$ $2(x^2+5)$ $\quad$ $4(x+5)^2$

(Total 1 mark)

Q8. $\mathrm{f}(x) = \frac{x}{x+2}$ $\quad$ $\mathrm{g}(x) = x^2 - 2$

Work out $\mathrm{fg}(x)$

Give your answer in the form $a + bx^n$ where a, b and n are integers.

(Total 3 marks)

Q9. For all values of x, $\mathrm{f}(x) = x^2 + 1$ $\quad$ $\mathrm{g}(x) = x - 5$

(a) Show that $\mathrm{fg}(x) = x^2 - 10x + 26$

(2)

(b) Solve $\mathrm{fg}(x) = \mathrm{gf}(x)$

(4)

(Total 6 marks)

Trigonometry

Q1. Circle the value of cos 30°

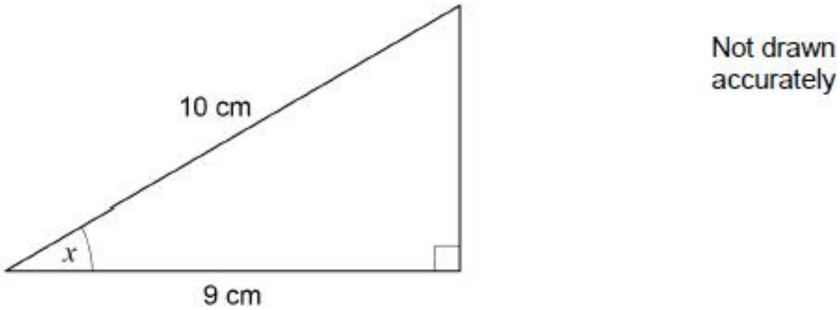
$\frac{1}{2}$ $\frac{\sqrt{3}}{2}$ 0 1

(Total 1 mark)

Q2. Show that the value of cos 30° × tan 60° + sin 30° is an integer.

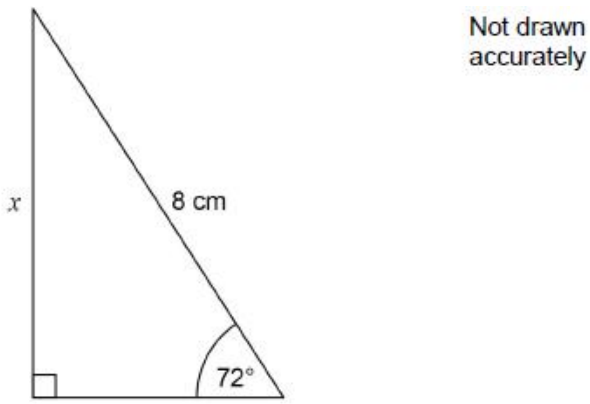
(Total 3 marks)

Q3. Use trigonometry to work out the size of angle x.



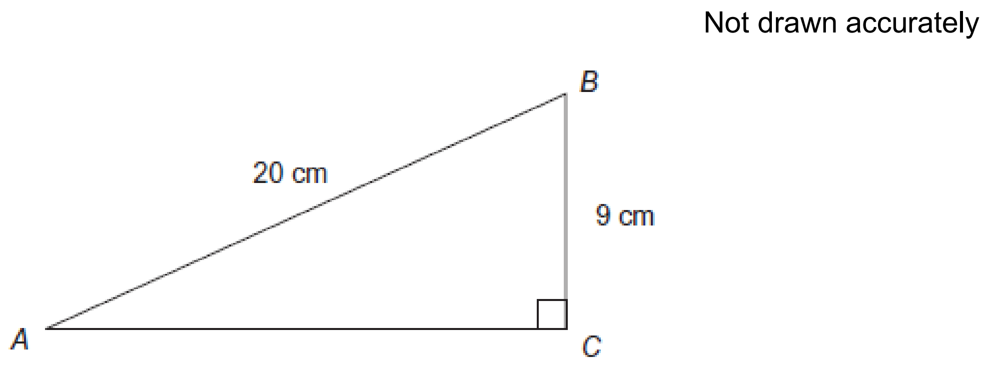
(Total 2 marks)

Q4. Use trigonometry to work out the length x.



(Total 2 marks)

Q5.

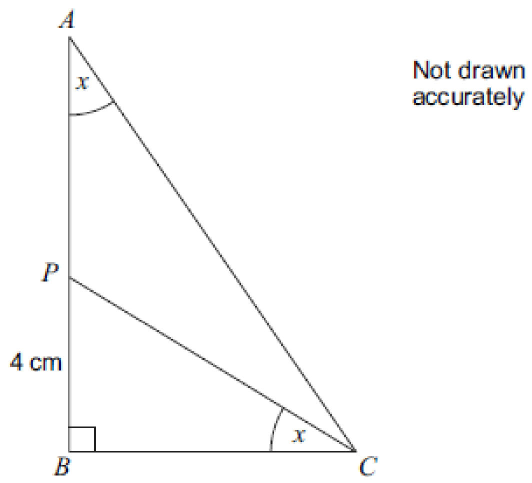


Work out the length AC.

(Total 3 marks)

Q6. ABC is a right-angled triangle.

P is a point on AB.



BP = 4cm and $\tan x = \frac{2}{3}$

(a) Work out the length of BC.

(2)

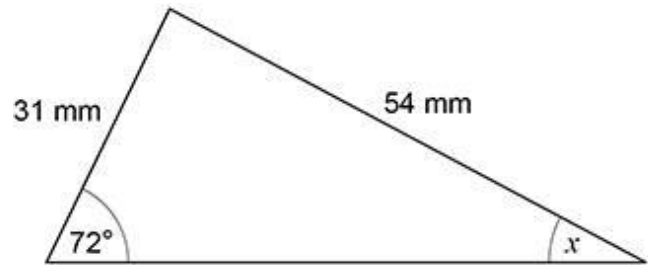
(b) Work out the length of AP.

(3)

(Total 5 marks)

Q7. Here is a triangle.

Not drawn accurately



Leah tries to use the sine rule to work out the size of angle x.

Here are the first two lines of her working.

$$\frac{x}{\sin 31} = \frac{54}{\sin 72}$$

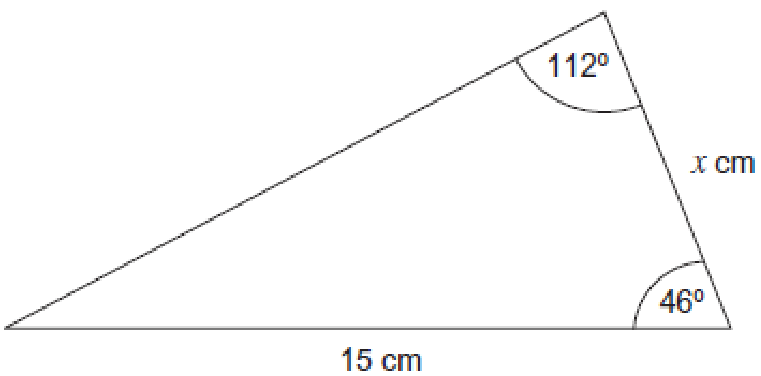
$$x = \frac{54 \sin 31}{\sin 72}$$

What error has she made in this working?

(Total 1 mark)

Q8.

Not drawn accurately



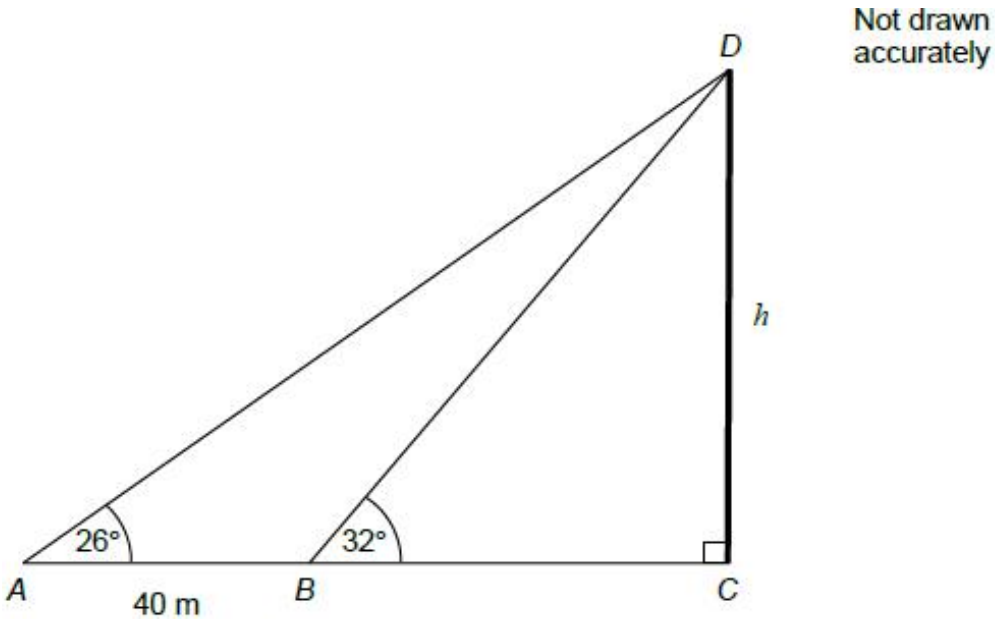
Work out the value of x.

(Total 4 marks)

Q9. The diagram shows a vertical tower *CD* of height, h, metres.

ABC is horizontal.

AB = 40 metres.



Work out the height, h, of the tower.

(Total 5 marks)

Quadratics

Q1. Circle the **two** roots of $(x - 5)(x + 3) = 0$

–5 –3 3 5

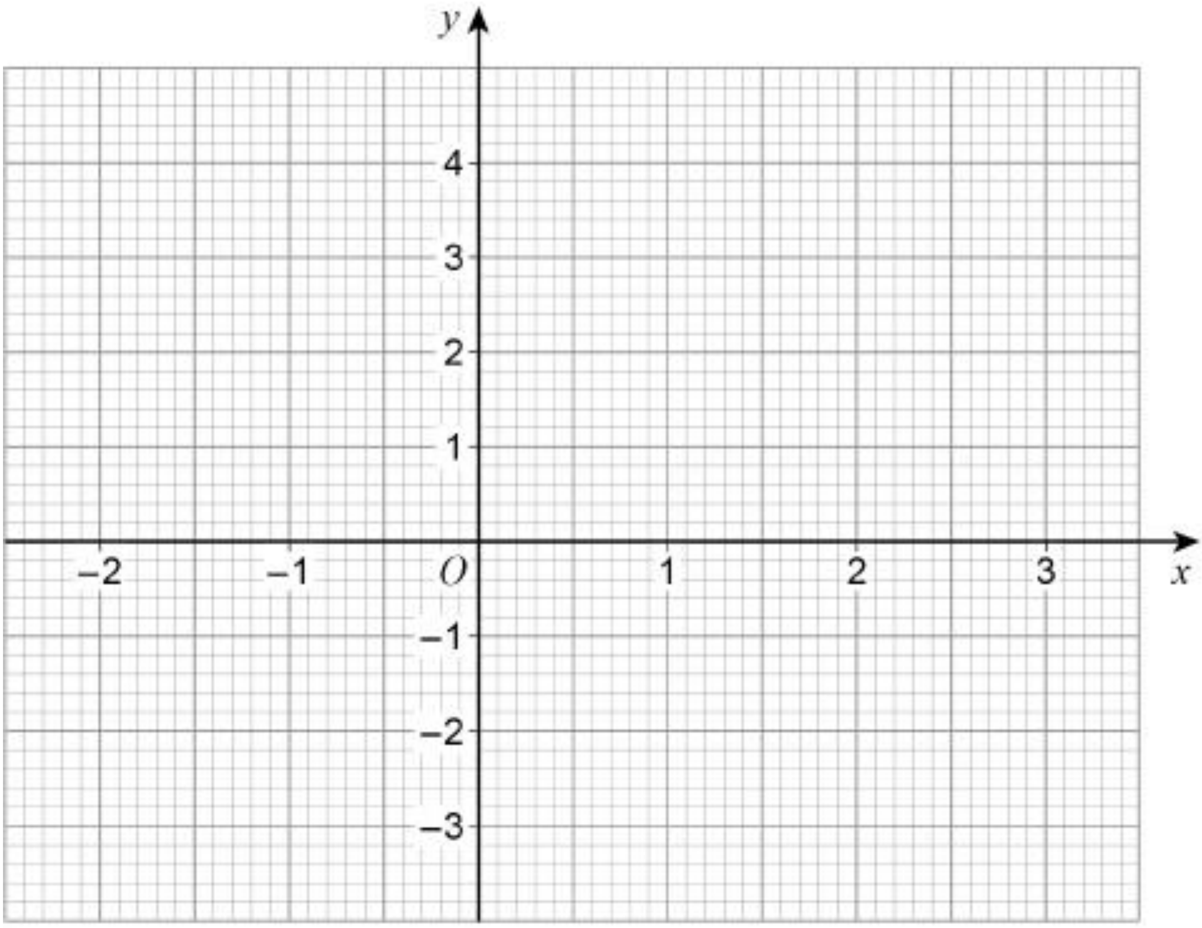
(Total 1 mark)

Q2. (a) Complete the table of values for $y = x^2 - x - 2$

x	−2	−1	0	1	2	3
y			−2	−2		4

(2)

(b) Draw the graph of $y = x^2 - x - 2$ for values of x from −2 to 3



(2)

(c) Write down the x-coordinate of the turning point of the graph.

(1)

(Total 5 marks)

Q3. The equation of a curve is $y = (x + 3)^2 + 5$

Circle the coordinates of the turning point.

(5, 3) (5, −3) (3, 5) (−3, 5)

(Total 1 mark)

Q4. (a) Write $x^2 + 6x + 10$ in the form $(x + a)^2 + b$

(2)

(b) Hence, write down the coordinates of the turning point of the curve $y = x^2 + 6x + 10$

(1)

(Total 3 marks)

Q5. (a) Factorise $x^2 + 5x - 24$

(2)

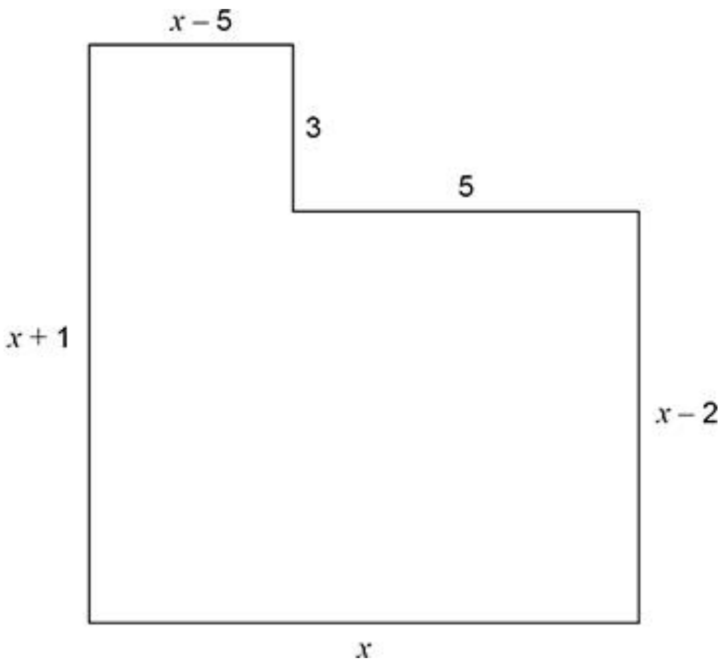
(b) Solve $x^2 + 5x - 24 = 0$

(1)

(Total 3 marks)

Q6. Here is the plan of the floor of an L-shaped room.

All lengths are in metres.



Not drawn accurately

(a) The area of the floor is 75 m²

Show that $x^2 + x - 90 = 0$

(3)

(b) By factorising $x^2 + x - 90$ work out the value of x.

You **must** show your working

(2)

(Total 5 marks)

Q7. The expression $\dfrac{x^2 - 9}{x^2 + bx - 15}$ simplifies to $\dfrac{x+3}{x+5}$

Work out the value of b.

(Total 3 marks)

Q8. Solve the quadratic equation

$$6x^2 + 2x - 5 = 0$$

Give your answers to 2 decimal places.

(Total 3 marks)

Q9. Solve the equation $\dfrac{5}{x+2} + \dfrac{4}{x+1} = 2$

(Total 6 marks)

Indices and Surds

Q1. Write down the value of 7^0

(Total 1 mark)

Q2. Given that $3^x = 9^{x+1}$ work out the value of x.

(Total 2 marks)

Q3. Simplify $2^5 \times 2^3$

Circle your answer.

4^8 2^8 2^{15} 4^{15}

(Total 1 mark)

Q4. Simplify $(5^4)^2$

Circle your answer.

5^6 5^8 25^6 25^8

(Total 1 mark)

Q5. Circle the value of $9^{-\frac{1}{2}}$

$\frac{1}{81}$ $\frac{1}{3}$ -3 $-4\frac{1}{2}$

(Total 1 mark)

Q6. Show that $\frac{14}{\sqrt{7}}$ can be written in the form $a\sqrt{b}$ where a and b are integers.

(Total 2 marks)

Q7. (a) Write $\sqrt{72}$ in the form $a\sqrt{2}$ where a is an integer.

(1)

(b) Work out $(\sqrt{6} + \sqrt{12})^2$

Give your answer in the form $c + d\sqrt{2}$ where c and d are integers.

(3)

(Total 4 marks)

Q8. Show that $\dfrac{\sqrt{150}-\sqrt{6}}{\sqrt{2}\times\sqrt{3}}$ simplifies to an integer.

(Total 3 marks)

Q9. Simplify $\sqrt{80}+\sqrt{2\frac{2}{9}}$

Give your answer in the form $\dfrac{a\sqrt{5}}{b}$ where a and b are integers.

(Total 3 marks)

Q10. Expand and simplify fully $(\sqrt{10}+\sqrt{2})(\sqrt{15}-\sqrt{3})$

Give your answer in the form $\sqrt{b}$, where a and b are integers.

(Total 4 marks)

Answers

Algebraic Manipulation and Proof

Q1. $6x^2 - 16xy + 15xy - 40y^2$

Allow one error

M1

$6x^2 - 16xy + 15xy - 40y^2$

Fully correct

A1

$6x^2 - xy - 40y^2$

ft their four terms

A1ft

[3]

Q2. **Alternative method 1 – multiplies $(x – 3)(x + 2)$ first**

$x^2 – 3x + 2x – 6$

or $x^2 – x – 6$

four terms with at least three correct

implied by $x^2 – x \pm k$ where k is a non-zero constant

M1

$x^3 – 3x^2 + 2x^2 – 6x + 5x^2 – 15x + 10x – 30$

or $x^3 – x^2 – 6x + 5x^2 – 5x – 30$

full expansion with correct multiplication of their 3 or 4 terms by x and 5

M1dep

$x^3 + 4x^2 – 11x – 30$

A1

Alternative method 2 – multiplies $(x – 3)(x + 5)$ first

$x^2 – 3x + 5x – 15$

or $x^2 + 2x – 15$

four terms with at least three correct

implied by $x^2 + 2x \pm k$ where k is a non-zero constant

1

$x^3 – 3x^2 + 5x^2 – 15x + 2x^2 – 6x + 10x – 30$

or $x^3 + 2x^2 – 15x + 2x^2 + 4x – 30$

full expansion with correct multiplication of their 3 or 4 terms by x and 2

M1dep

$x^3 + 4x^2 – 11x – 30$

A1

Alternative method 3 – multiplies $(x + 2)(x + 5)$ first

$x^2 + 2x + 5x + 10$

or $x^2 + 7x + 10$

four terms with at least three correct

implied by $x^2 + 7x \pm k$ where k is a non-zero constant

M1

$x^3 + 2x^2 + 5x^2 + 10x – 3x^2 – 6x – 15x – 30$

or $x^3 + 7x^2 + 10x – 3x^2 – 21x – 30$

full expansion with correct multiplication of their 3 or 4 terms by x and –3

M1dep

$x^3 + 4x^2 – 11x – 30$

A1

Additional Guidance

Do not ignore further incorrect simplification or attempt to solve after correct answer seen

[3]

Q3. $y(x − 5) = 4 − 3x$

M1

$xy − 5y = 4 − 3x$

M1

$xy + 3x = 4 + 5y$

or $x(y + 3) = 4 + 5y$

Isolating x terms

M1

$x = \frac{4 + 5y}{y + 3}$

oe

A1

[4]

Q4. $(x − 3)^2 \equiv x^2 − 6x + 9$

B1

[1]

Q5. Alternative method 1

$35x + 6x = ax$ or $35 + 6 = a$

or $41x = ax$

M1

$a = 41$

A1

$40 + 3b = 13$

oe

M1

$b = −9$

SC3 $a = 41$, $b = –27$ *or* $a = 41$, $b = \frac{5}{3}$

A1

Alternative method 2

$35x + 40 + 6x + 3b$ or $41x + 40 + 3b$

M1

$35x + 6x = ax$ or $35 + 6 = a$

and

$40 + 3b = 13$

oe *eg* $41x = ax$ *and* $3b = −27$

M1dep

$a = 41$

implies first M1 only

A1

$b = −9$

SC3 $a = 41$, $b = −27$ *or* $a = 41$, $b = \frac{5}{3}$

A1

Additional Guidance

a = 41 and b = −9

M1A1M1A1

a = 41 or b = −9

M1A1

35x, 40, 6x and 3b seen without addition signs shown or implied

M0

35x + 40 + 6x + b leading to an answer of a = 41 and b = −27

SC3

35x + **8** + 6x + 3b leading to an answer of a = 41 and b = $\frac{5}{3}$

SC3

35x + **8** + 6x + b leading to an answer of a = 41 and b = 5

M1A1

a = 41x

M0

For $\frac{5}{3}$ accept 1.66... or 1.67

Condone multiplication signs eg 35 × x for 35x

[4]

Q6. $x^2 + x + 7x + 7$

or $x^2 + 8x + 7$

oe

M1

$3x^2 + 24x + 21$

M1dep

$4x^2 + 10x + 10x + 25$

or $4x^2 + 20x + 25$

oe

M1

$-x^2 + 4x - 4$

oe

A1

$-(x - 2)^2$ so never positive

A1

[5]

Q7. Alternative method 1

$\frac{n^2+n}{2}$ or $\frac{n^2+2n+n+2}{2}$

or $\frac{n^2+3n+2}{2}$

may be seen in stages

e.g. $n^2 + n$ *followed by* $\frac{n^2+n}{2}$

M1

$\frac{n^2+n}{2}$ and $\frac{n^2+2n+n+2}{2}$

or

$\frac{n^2+n}{2}$ and $\frac{n^2+3n+2}{2}$

may be seen in stages

e.g. $n^2 + n$ *followed by* $\frac{n^2+n}{2}$

and $n^2 + 3n + 2$ *followed by* $\frac{n^2+3n+2}{2}$

implies M2

$\frac{2n^2+4n+2}{2}$ or $n^2 + 2n + 1$

with M2 seen

oe single fraction with terms collected

e.g. $\frac{4n^2+8n+4}{4}$

A1

$n^2 + 2n + 1$ and $(n + 1)^2$

with M2A1 seen

allow $(n + 1)(n + 1)$ *for* $(n + 1)^2$

A1

Alternative method 2

$\frac{n+1}{2}(n+n+2)$

oe e.g. $(n+1)(\frac{n}{2}+\frac{n+2}{2})$

M1

$\frac{n+1}{2}(2n+2)$

or $\frac{n^2+n}{2}+\frac{n^2+n}{2}+\frac{2n+2}{2}$

with M1 seen

M1dep

$\frac{2n^2+4n+2}{2}$ or $n^2 + 2n + 1$

with M2 seen

oe single fraction with terms collected

e.g. $\frac{4n^2+8n+4}{4}$

A1

$n^2 + 2n + 1$ and $(n + 1)^2$

with M2A1seen

allow $(n + 1)(n + 1)$ *for* $(n + 1)^2$

A1

Alternative method 3

$\frac{n+1}{2}(n+n+2)$

oe eg $(n+1)(\frac{n}{2}+\frac{n+2}{2})$

M1

$\frac{n+1}{2}(2n+2)$ with M1 seen

oe eg $\frac{(n+1)(2n+2)}{2}$

M1dep

$(n + 1)^2$ with M2 seen

A1 $2(n+1)\frac{n+1}{2}$ *or* $\frac{2(n+1)^2}{2}$

allow $(n + 1)(n + 1)$ *for* $(n + 1)^2$

A2

Additional Guidance

Only substituting in values of n

M0M0A0A0

Consistently using a different letter to n can score up to M1M1A1A1

Using two different letters consistently within the two fractions (e.g. n replaced by x in the first equation and n replaced by y in the second equation) can score a maximum of M1M1A0A0 unless recovered to the same letter

Multiplying fractions instead of adding can score a maximum of M2A0

For M marks condone e.g. $n2$ for $2n$ etc

$n^2 + n$ /2 and $n^2 + 3n + 2$/2 recovered to $\frac{2n^2+4n+2}{2}$

and/or $n^2 + 2n + 1$ and/or $(n + 1)^2$

M1M1A0A0

$n^2 + n$ /2 and $n^2 + 3n + 2$/2 not recovered

M0M0A0A0

$n^2 + n$ and $n^2 + 3n + 2$ recovered to $\frac{2n^2+4n+2}{2}$

and/or $n^2 + 2n + 1$ and/or $(n + 1)^2$

M1M1A0A0

$n^2 + n$ and $n^2 + 3n + 2$ not recovered

M0M0A0A0

Equating to n^2 in working can score a maximum of M1M1A0A0

(equating to e.g. x^2 can score up to M1M1A1A1)

$1n$ is allowed for n throughout

Alts 2 and 3

$\frac{n+1}{2}(2n+2)$ with M1 seen scores M2

If they attempt to expand $(n + 1)(2n + 2)$ use Alt 2

If they attempt to expand $\frac{1}{2}(2n + 2)$ use Alt 3

[4]

Q8. $(n + 3)^2 - n^2$

$n^2 - (n - 3)^2$

M1

$n^2 + 3n + 3n + 9 - n^2$

$(= 6n + 9)$

$n^2 - n^2 + 3n + 3n - 9$ $(= 6n - 9)$

A1

$3[n + (n + 3)]$

$3[n + (n - 3)]$

A2

Complete solution with all stages clearly shown

Strand (ii)

Alternative method

$x^2 - y^2 = (x + y)(x - y)$

Must see difference of two squares factorisation

M1

$x - y = 3$

M1 dep

$x^2 - y^2 = (x + y).3$

A2

Complete solution with all stages clearly shown

Strand (ii)

[4]

Straight Lines and Circles

Q1. $y = 3x$

B1

[1]

Q2. $\frac{3}{2}$

B1

[1]

Q3. (a) **Alternative method 1 – Using gradients**

Gradient of $y = 3x + 7$ is 3

and $y = 3x + 4$

and

gradient of $2y - 6x = 8$ is 3 or 6 ÷ 2

B3

May come from using points on line
eg using (0, 7) and (1, 10)
and $\frac{10-7}{1-0} = 3$
or correct calculation for gradient from points on line
$2y - 6x = 8$
eg using (0, 4) and (1, 7) and $\frac{7-4}{1-0} = 3$
B2 for $y = 3x + 4$ and lines have same gradient
or $y = 3x + 4$
and gradient of $2y - 6x = 8$ is 3 or 6 ÷ 2
or gradient of $y = 3x + 7$ is 3
and $y = 3x + 4$
B1 for gradient of $y - 3x + 7$ is 3
or $y = 3x + 4$
or gradient of $2y – 6x = 8$ is 3 or 6 ÷ 2

Alternative method 2 – Using coordinates and distances

Choose a value for x and correctly evaluates the y value for both lines

eg (0, 7) and (0, 4)

M1

Chooses a different value for x and correctly evaluates the y value for both lines

eg (1, 10) and (1, 7)

M1dep

States that y values are a constant distance apart so parallel

oe

A1

Alternative method 3 – Using simultaneous equations

$y = 3x + 4$

or $y - 3x = 4$

or $2y = 6x + 14$

or $2y - 6x = 14$

oe

Equates coefficients in any form

M1

Any attempt to eliminate both variables from their equations

M1dep

States simultaneous equations have no (real) solution and concludes parallel

A1

Additional Guidance

To award A mark on Alternative method 2, the working must be seen

$y = 3x + 4$ and lines have gradient of $3x$

B2

$y = 3x + 4$ and $3x$ identified in both equations

B2

Both lines have gradient $3x$

B1

$y = 3x + 7$, gradient 3 and $y = 3x + 8$, gradient 3 (error in rearrangement)

B1

$y = 3x + 8$, gradient 3 (error in rearrangement)

B0

Parallel as both have same gradient

B0

$2(3x + 7) - 6x = 8$

M1

$6x + 14 - 6x = 8$

$14 = 8$

M1

$y = 3x + 7$ and $y = \frac{8 + 6x}{2}$ are equated coefficients,

M1

(b) $3 \times -5 + 7$

or $-15 + 7$

or -8

or $(-5, -8)$

Use a point on y = 3x + 7 with (–5, –6) to compare gradient to 3
eg Gradient from (–5, –6) to (0, 7) is 2.6

or $(-6 - 7) \div 3$ or $-4.33...$

or $y = 3x + 9$

M1

Above and –8

or Above and –4.33

or Above and $y = 3x + 9$

oe
Above and eg Gradient from (–5, –6) to (0, 7) is 2.6

A1

Additional Guidance

Do not ignore incorrect statements eg –6 is less than –8 so above

M1A0

(0, 7), (–1, 4), (–2, 1), (–3, –2), (–4, –5), (–5, –8) and ticks below

M1A0

[5]

Q4. $-\frac{1}{4}$ or –1 ÷ 4

oe

M1

5 = their $-\frac{1}{4} \times 8 + c$ or $c = 7$

or

$y - 5 = -\frac{1}{4}(x - 8)$

oe

$y = -\frac{1}{4}x + 7$ *implies M2*

M1dep

$-\frac{1}{4}x + 7 = 0$ or (x =) 28

oe

M1dep

(28, 0)

SC2 (–12, 0) or (6.75, 0)

A1

Additional Guidance

Answer (0, 28) is A0 but may score M marks if working seen

(–12, 0) from using the gradient of the perpendicular as $\frac{1}{4}$

SC2

(6.75, 0) from using the gradient of the perpendicular as 4

SC2

[4]

Q5. 2

B1

[1]

Q6. 6

B1

[1]

Q7. (a) $(0^2 +)\ 6^2 = 36$

or (OA =) radius = 6

or $\sqrt{36} = 6$

oe

B1

Additional Guidance

0 + 36 = 36

B0

(b) (6, 0)

B1

(c) **Alternative method 1**

$\frac{6 - \text{their } 0}{0 - \text{their } 6}$ or $\frac{\text{their } 0 - 6}{\text{their } 6 - 0}$

or $\frac{6}{-6}$ or $\frac{-6}{6}$ or -1

gradient AB

M1

gradient OM × gradient AB = −1

and

gradient OM = 1 (and $y = x$)

must see correct working for M1

A1

Alternative method 2

$\left(\frac{6+0}{2}, \frac{0+6}{2}\right)$ or (3, 3)

coordinates of M

M1

gradient OM = 1 (and $y = x$)

or (0, 0) and (3, 3) (and $y = x$)

must see correct working for M1

A1

(d) $x^2 + x^2 = 36$ or $2x^2 = 36$

or $y^2 + y^2 = 36$ or $2y^2 = 36$

or (−)6 cos 45° or (−)6 sin 45°

oe equation

M1

$(-)\sqrt{\frac{36}{2}}$ or $(-)\sqrt{18}$ or $(-)3\sqrt{2}$

or $(-)\frac{6\sqrt{2}}{2}$ or $(-)\frac{6}{\sqrt{2}}$

M1

$(-\sqrt{18}, -\sqrt{18})$ or $(-3\sqrt{2}, -3\sqrt{2})$

or $\left(-\frac{6\sqrt{2}}{2}, -\frac{6\sqrt{2}}{2}\right)$

or $\left(-\frac{6}{\sqrt{2}}, -\frac{6}{\sqrt{2}}\right)$

oe surd form

A1

[7]

Q8. (a) $0^2 + (-5)^2 = 25$

or

$(0^2 +)\ y^2 = 25$ and $y = -5$

or

radius of the circle is 5

oe

B1

Additional Guidance

B is on the y-axis

B0

B is below the Origin

B0

(b) Tangents from an external point are

equal in length

B1

(c) **Alternative method 1**

$(a - 3)^2 + (-5 - 4)^2 = a^2$

oe

$PA^2 = PB^2$

M1

$a^2 - 3a - 3a + 9 + 81 = a^2$

Expands brackets

Allow one error

M1

$6a = 90$

Rearranges their quadratic equation

to $ka = c$

M1

$(a =)$ 15 and $(b =)$ −5

SC1 $b = -5$

A1

Alternative method 2

(grad *AP* =) $-1 \div \frac{4}{3}$ or $-\frac{3}{4}$

M1

$y - 4 = -\frac{3}{4}(x - 3)$

or

$y = -\frac{3}{4}x + c$ and substitutes (3, 4)

oe eg $y = -\frac{3}{4}x + \frac{25}{4}$

ft their gradient AP

M1

$-5 = -\frac{3}{4}x + \frac{25}{4}$

oe

ft their equation AP

M1

$(a =)$ 15 and $(b =)$ −5

SC1 $b = -5$

A1

Alternative method 3

$$\frac{XP}{4- -5} = \frac{4}{3}$$

oe

May be seen on diagram

X is foot of perpendicular from A to BP

M1

$XP = \frac{4}{3} \times (4 - -5)$ or $XP = 12$

oe

Must have calculation or value for XP

M1

12 + 3 or 15

oe

M1

(a =) and (b =) −5

SC1 b = −5

A1

[6]

Functions

Q1. 73

B1

Additional Guidance

Mark output box if answer line blank

[1]

Q2. 36

B1

[1]

Q3. $3x^2$

B1

[1]

Q4. (a) 19

B1

(b) + 7 and × 2

Must be in correct order

B1

(c) $y = \frac{x}{2} + 3$

B1

[3]

Q5. (a) −4 **and** ×3 or ×3 **and** −12

Must be in the correct order for B2

B1 for −4 or n − 4 in first box, or ×3 or 3 × n in first box

***Note**: ×3 **and** −4 scores B0*

*B1 for 3 **and** −12 (missing × sign)*

B2

(b) $3(n - 4) = n$ or $3n - 12 = n$

M1

6

A1

[4]

Q6. $5f(x) = 4x - 3$ or $5f(x) + 3 = 4x$

or $5y = 4x - 3$ or $5y + 3 = 4x$

or $5x = 4y - 3$ or $5x + 3 = 4y$

Accept any letter used for y

M1

$\frac{5f(x)+3}{4}\ (= x)$

or $\frac{5y+3}{4}\ (= x)$

M1

$\frac{5x + 3}{4}$

Condone y = (or any other letter)

A1

[3]

Q7. $2(x + 5)^2$

B1

[1]

Q8. $\frac{x^2-2}{x^2-2+2}$ or $\frac{x^2-2}{x^2}$

M1

$\frac{x^2}{x^2} - \frac{2}{x^2}$ or $1 - \frac{2}{x^2}$

implied by correct final answer
must be two terms
oe eg $x^2x^{-2} - 2x^{-2}$

A1

$1 - 2x^{-2}$

or

$a = 1$ and $b = -2$ and $n = -2$

A1

[3]

Q9. (a) $(x - 5)^2 + 1$

M1

$x^2 - 5x - 5x + 25 + 1$

$= x^2 - 10x + 26$

A1

(b) $x^2 + 1 - 5$ or $x^2 - 4$

B1

$x^2 - 10x + 26 =$ their $(x^2 - 4)$

M1

$-10x = -4 - 26$

or $-10x = -30$

or $10x = 30$

oe

M1

3

A1

[6]

Trigonometry

Q1. $\frac{\sqrt{3}}{2}$

B1

[1]

Q2. $\frac{\sqrt{3}}{2} \times \sqrt{3} + \frac{1}{2}$

$= \frac{3}{2} + \frac{1}{2}$

= 2

B2 $\frac{\sqrt{3}}{2} \times \sqrt{3} + \frac{1}{2}$

B1 cos 30° = $\frac{\sqrt{3}}{2}$ *or tan 60°* = $\sqrt{3}$

or sin 30° = $\frac{1}{2}$

B3

Additional Guidance

For B3 all steps must be shown

Allow $\frac{\sqrt{3}}{2} \times \sqrt{3} + \frac{1}{2}$ given as $\frac{\sqrt{3}}{2} \times \sqrt{3}$, followed by their $\frac{3}{2} + \frac{1}{2}$

Allow equivalent expressions for all trig values

eg

cos 30° = $\sqrt{\frac{3}{4}}$ sin 30° = $\frac{\sqrt{1}}{2}$ tan 60°=$\frac{\sqrt{3}}{\sqrt{1}}$

For B1 allow the trig value(s) given in a table unless contradicted in working

[3]

Q3. cos x = $\frac{9}{10}$

oe

eg

sin x = $\frac{\sqrt{10^2 - 9^2}}{10}$

tan x = $\frac{\sqrt{10^2 - 9^2}}{9}$

M1

25.8... or 26

A1

Additional Guidance

cos = $\frac{9}{10} x$ = 25.8 (recovered)

M1A1

cos = $\frac{9}{10}$

M0A0

[2]

Q4. $\sin 72 = \frac{x}{8}$

or 8 × sin 72

or $\cos(90 - 72) = \frac{x}{8}$

or 8 × cos (90 – 72)

or $\frac{x}{sin72} = \frac{8}{sin90}$

or $\frac{sin72}{x} = \frac{sin90}{8}$

oe

eg 8 cos 72 or 2.47… or 2.5 ***and*** $\sqrt{8^2 - (8cos72)^2}$

M1

[7.6, 7.61]

A1

Additional Guidance

If trigonometry and Pythagoras are used it must be a fully correct method that would lead to the correct value of x

Accept sin 72 × 8

M1

Accept opp or o for x eg $sin72 = \frac{\text{opp}}{8}$

M1

$\sin = \frac{x}{8}$ or $\sin\theta = \frac{x}{8}$ (unless recovered)

M0

Answer coming from scale drawing

M0A0

Answer in range seen followed by 7 or 8

M1A1

[2]

Q5. 20^2 and 9^2

or 400 and 81

or 319

oe

M1

$\sqrt{20^2 - 9^2}$

or $\sqrt{400 - 81}$

or $\sqrt{319}$

M1dep

17.86… or 17.9

Accept 18 if working shown

A1

[3]

Q6. (a) $\frac{4}{BC} = \frac{2}{3}$

oe

M1

(BC =) 6

A1

(b) $\frac{their\ 6}{AB} = \frac{2}{3}$

oe

e.g. follow through their 6 using a similar triangles / scale factor method

M1

(AB =) 9

A1ft

(AP =) 5

A1ft

[5]

Q7. Correct explanation

eg (it should be) $\frac{31}{sinx}$

B1

Additional Guidance

x and 31 should be swapped

B1

She has used 31 as an angle

B1

She has used x as a length

B1

It should be $\frac{sinx}{31}$ $(= \frac{sin72}{54})$

B1

[1]

Q8. 180 − 112 − 46 or 22

May be seen on the diagram

M1

$\frac{15}{sin112} = \frac{x}{\sin their\ 22}$

oe

M1

$\frac{15\ \sin their\ 22}{sin112}$

M1

6.06… or 6.1 or 6

A1

[4]

Q9. **Alternative method 1**

$$\frac{AD}{\sin(180-32)} = \frac{40}{\sin(32-26)}$$

M1

$\frac{40}{\sin(32-26)} \times \sin(180-32)$

or 202.7... or 202.8

M1dep

sin 26 = $\frac{h}{their\ 202.8}$

M1

their 202.8 × sin 26

M1

[88.89, 88.9] or 89

A1

Alternative method 2

$$\frac{BD}{\sin 26} = \frac{40}{\sin(32-26)}$$

M1

$\frac{40}{\sin(32-26)}$ × sin 26

or 167.7... or 167.8

M1dep

sin 32 = $\frac{h}{their\ 167.8}$

M1

their 167.8 × sin 32

M1

[88.89, 88.9] or 89

A1

Alternative method 3

BC tan 32 = (BC + 40) tan 26
oe

M1

(BC =) $\frac{40tan26}{tan32-tan26}$

or 142.26... or 142.3

M1dep

(AC =) their 142.26... + 40

or 182.26... or 182.3

tan 32 = $\frac{h}{142.26}$

M1

their 182.26... × tan 26

or their 142.26 × tan 32

M1

[88.89, 88.9] or 89

A1

Alternative method 4

h = BC tan 32 and h = (BC + 40) tan 26
oe

M1

$h = \left(\frac{h}{tan32} + 40\right) tan26$
Using BC = $\frac{h}{tan32}$

M1dep

h tan 32 = h tan 26 + 40 tan 26 tan 32

M1

$(\frac{40tan26tan32}{tan32-tan26})$

M1

[88.89, 88.9] or 89

A1

[5]

Quadratics

Q1. –3 and 5

B1

[1]

Q2. (a)

x	−2	−1	0	1	2	3
y	4	0	−2	−2	0	4

B1 1 or 2 values correct

B2

(b) 5 or 6 points plotted correctly

Correct or ft their table in (a)

Tolerance of ±1 small square

Points can be implied by graph passing through them

M1

Correct smooth parabolic curve

*Tolerance of ±1 small square for the six **correct** points from the table*

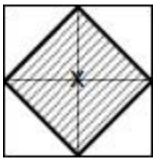
and y-coordinate of minimum point in the range $-2.5 \le y \le -2.1$

No further tolerance for the minimum

A1

Additional Guidance

Tolerance of ±1 small square means it is on the edges of or within the shaded area



Ignore extra points plotted

If their table in (a) has points that are beyond the grid these points will not be able to be plotted correctly

Ignore any curve drawn for $x < -2$ or $x > 3$

Curve passing through all correct points within tolerance

M1A1

Ruled straight lines

A0

(c) $\frac{1}{2}$ or 0.5

Ignore any y-coordinate

B1

Additional Guidance

(–2.25, 0.5)

B0

Ignore their graph drawn in (b) – there is no ft

Condone 0.5, –2.25

B1

[5]

Q3. (−3, 5)

B1

[1]

Q4. (a) **Alternative method 1**

$(x + 3)^2 + \ldots$ or $a = 3$

M1

$(x + 3)^2 + 1$

Accept $a = 3$ and $b = 1$

A1

Alternative method 2

$2a = 6$ and $a^2 + b = 10$

M1

$(x + 3)^2 + 1$

Accept $a = 3$ and $b = 1$

A1

(b) (−3, 1)

oe

ft their a and their b

B1ft

[3]

Q5. (a) $(x + a)(x + b)$

where $ab = \pm 24$

M1

$(x + 8)(x - 3)$

either order

A1

(b) (x =) –8 and (x =) 3

ft their part (a)

B1 ft

[3]

Q6. (a) **Alternative method 1 – horizontal split**

$x(x - 2)$ and $3(x - 5)$

oe may be seen as two areas

M1

$x^2 - 2x + 3x - 15$ (= 75)

oe expression with all brackets expanded

M1dep

$x^2 - 2x + 3x - 15 = 75$

and $x^2 + x - 90 = 0$

or

$x^2 + x - 15 = 75$

and $x^2 + x - 90 = 0$

with full working seen

A1

Alternative method 2 – vertical split

$(x - 5)(x + 1)$ and $5(x - 2)$

oe may be seen as two areas

M1

$x^2 - 5x + x - 5 + 5x - 10$ (= 75)

or

$x^2 - 4x - 5 + 5x - 10$ (= 75)

oe expression with all brackets expanded

M1dep

$x^2 - 5x + x - 5 + 5x - 10 = 75$

and $x^2 + x - 90 = 0$

or

$x^2 - 4x - 5 + 5x - 10 = 75$

and $x^2 + x - 90 = 0$

with full working seen

A1

Alternative method 3 – large rectangle subtract 3 × 5

$x(x + 1)$ and 3×5
oe may be seen as two areas
M1

$x^2 + x - 15$ (= 75)
oe expression with brackets expanded and 3 × 5 evaluated
M1dep

$x^2 + x - 15 = 75$

and $x^2 + x - 90 = 0$
with full working seen
A1

Alternative method 4 – split into three areas

$3(x - 5)$ and $(x - 2)(x - 5)$ and $5(x - 2)$
oe may be seen as three areas
M1

$3x - 15 + x^2 - 2x - 5x + 10 + 5x - 10$ (= 75)

or

$3x - 15 + x^2 - 7x + 10 + 5x - 10$ (= 75)
oe expression with all brackets expanded
M1dep

$3x - 15 + x^2 - 2x - 5x + 10 + 5x - 10 = 75$

and $x^2 + x - 90 = 0$

or

$3x - 15 + x^2 - 7x + 10 + 5x - 10 = 75$

and $x^2 + x - 90 = 0$
with full working seen
A1

Additional Guidance

Ignore attempts to solve the equation or substituting values for x

Condone missing end bracket for M1

Condone missing pairs of brackets if recovered

eg $3 \times x - 5$ recovered to $3x - 15$

(b) $(x - 9)(x + 10)$ (= 0)

and answer 9

B1 $(x - 9)(x + 10)$ (= 0)

and answer 9 and –10

SC1 $(x + 9)(x - 10)$ (= 0)

and answer 10

B2

Additional Guidance

If no response is seen, check part (a) for any creditworthy work

Answer 9 with no working can be awarded up to B2 from correct factorising seen in part (a)

Answer 9 from quadratic formula or completing the square

B1

Answer 9 and –10 from quadratic formula or completing the square

B0

Answer from trial and improvement only

B0

[5]

Q7. $(x - 3)(x + 3)$

Substitutes any value for x into both expressions but not $x = 0$

M1

$(x - 3)(x + 5)$

Sets up a correct equation in b

M1dep

(b =) 2 or $x^2 + 2x - 15$

A1

[3]

Q8. $(x =) \frac{-2 \pm \sqrt{(2)^2 - 4(6)(-5)}}{2(6)}$

Allow one error

M1

$(x =) \frac{-2 \pm \sqrt{(2)^2 - 4(6)(-5)}}{2(6)}$

$(x =) \frac{-2 \pm \sqrt{124}}{12}$

A1

0.76 and −1.09

M1

[3]

Q9. $5(x + 1)$ or $4(x + 2)$

or $(x + 2)(x + 1)$

or $2(x + 2)(x + 1)$

oe

M1

$5x + 5 + 4x + 8$

or $x^2 + 2x + x + 2$

or $x^2 + 3x + 2$

or $2x^2 + 4x + 2x + 4$

or $2x^2 + 6x + 4$

Allow 1 error

M1dep

their $5x + 5 + 4x + 8 = 2(x + 2)(x + 1)$

oe

M1dep

$2x^2 - 3x - 9 = 0$

or $2x^2 - 3x = 9$

or $2x^2 = 3x + 9$

Correctly simplified to three terms

A1

$(2x + 3)(x - 3)$

Attempt to factorise their quadratic or uses quadratic formula with at most one error

i.e. $(mx + a)(nx + b)$ where mn = their 2 and ab = ± their 9

M1

$x = -\frac{3}{2}$ and $x = 3$

A1

[6]

Indices and Surds

Q1. 1

B1

[1]

Q2. (x =) 2(x + 1) or 2x + 1

or $\frac{1}{2}x\ (= x + 1\ )$

oe May be seen as an index is $(3^2)^{x+1}$ *or* $9^{\frac{1}{2}x}$

M1

–2

Correct answer is 2 marks even if working nonsense or wrong.

A1

[2]

Q3. 2^8

B1

[1]

Q4. 5^8

B1

[1]

Q5. $\frac{1}{3}$

B1

[1]

Q6. $\frac{14}{\sqrt{7}} \times \frac{\sqrt{7}}{\sqrt{7}}$ or $\frac{14\sqrt{7}}{\sqrt{7}}$

M1

$2\sqrt{7}$

do not award if further work eg $\sqrt{14}$

A1

Additional Guidance

Correct answer with no working

M1A1

[2]

Q7. (a) $6\sqrt{2}$

B1

(b) $(\sqrt{6})^2 + \sqrt{6} \times \sqrt{12} + \sqrt{6} \times \sqrt{12} + (\sqrt{12})^2$

oe any expansion with 4 correct terms implied

M1

$6 + \sqrt{72} + \sqrt{72} + 12$

oe eg $\sqrt{36} + 2\sqrt{72} + \sqrt{144}$

A1

$18 + 12\sqrt{2}$

ft 18 + 2 × their (a) for $\sqrt{2}$ *term*

A1ft

Alternative method

$(\sqrt{6})^2(1 + \sqrt{2})^2$

M1

$6(1 + 2\sqrt{2} + 2)$

A1

$18 + 12\sqrt{2}$

A1ft

[4]

Q8. Alternative method 1

$(\sqrt{150} =)\ \sqrt{25}\sqrt{6}$ or $5\sqrt{6}$

numerator allow $\sqrt{2}\sqrt{3}$ for $\sqrt{6}$

or

$\left(\sqrt{2} \times \sqrt{3} = \right) \sqrt{6}$

denominator

M1

$\frac{\sqrt{25}\sqrt{6} - \sqrt{6}}{\sqrt{6}}$ or $\frac{5\sqrt{6} - \sqrt{6}}{\sqrt{6}}$ or $\frac{4\sqrt{6}}{\sqrt{6}}$

allow consistent use of $\sqrt{2}\sqrt{3}$ for $\sqrt{6}$

M1dep

4 with M1M1 awarded

A1

Alternative method 2

$\sqrt{6}(\sqrt{25} - 1)$ or $\sqrt{6}(5 - 1)$

numerator allow $\sqrt{2}\sqrt{3}$ for $\sqrt{6}$

or $4\sqrt{6}$

or

$\left(\sqrt{2} \times \sqrt{3} = \right) \sqrt{6}$

denominator

M1

$\frac{\sqrt{6}(\sqrt{25} - 1)}{\sqrt{6}}$ or $\frac{\sqrt{6}(5-1)}{\sqrt{6}}$

allow consistent use of $\sqrt{2}\sqrt{3}$ for $\sqrt{6}$

M1dep

4 with M1M1 awarded

A1

Alternative method 3

$\frac{\sqrt{150} - \sqrt{6})}{\sqrt{2} \times \sqrt{3}} \times \frac{\sqrt{6}}{\sqrt{6}}$

allow $\frac{\sqrt{2}\sqrt{3}}{\sqrt{2}\sqrt{3}}$ *for* $\frac{\sqrt{6}}{\sqrt{6}}$

M1

$\frac{\sqrt{900} - 6}{6}$

oe rationalised

M1dep

4 with M1M1 awarded

A1

Additional Guidance

Condone answer 4 and –6 from use of $\sqrt{25} = \pm 5$

M1M1A1

[3]

Q9. $\frac{14\sqrt{5}}{3}$

oe eg $\frac{28\sqrt{5}}{6}$

B2 $\left(\sqrt{2\frac{2}{9}} = \right)\frac{2\sqrt{5}}{3}$

or

$(\sqrt{80} =)\ 4\sqrt{5}$ *and*

$\left(\sqrt{2\frac{2}{9}} = \right)\frac{\sqrt{20}}{3}$ *or* $\left(\sqrt{2\frac{2}{9}} = \right)\frac{2\sqrt{5}}{\sqrt{9}}$

B1 $(\sqrt{80} =)\ 4\sqrt{5}$

or $\left(\sqrt{2\frac{2}{9}} = \right)\frac{\sqrt{20}}{3}$ *or* $\left(\sqrt{2\frac{2}{9}} = \right)\frac{2\sqrt{5}}{\sqrt{9}}$

B3

Additional Guidance

For B1 or B2, allow $\frac{6\sqrt{5}}{9}$ for $\frac{2\sqrt{5}}{3}$ and $\frac{\sqrt{180}}{9}$ for $\frac{\sqrt{20}}{3}$

$\frac{14}{3}\sqrt{5}$

B3

$16\sqrt{5} + \frac{2\sqrt{5}}{3} = \frac{50\sqrt{5}}{3}$

B2

$4\sqrt{5} + \frac{2\sqrt{5}}{3} = 4\frac{2}{3}\sqrt{5}$

B2

$4\sqrt{5} + \frac{2\sqrt{5}}{9} = \frac{38\sqrt{5}}{9}$

B1

$2\sqrt{20} + \frac{\sqrt{20}}{3} = \frac{7\sqrt{20}}{3}$

B1

[3]

Q10. $\sqrt{10}\sqrt{15} - \sqrt{10}\sqrt{3}\ (+)\sqrt{2}\sqrt{15} - \sqrt{2}\sqrt{3}$

or better …

Allow one error (sign or term) in the expansion

M1

Eliminating the two 'middle' terms

These must be the correct two middle terms

M1

$\sqrt{10}\sqrt{15}$ simplified to $5\sqrt{6}$

M1

$4\sqrt{6}$

A1

Alternative method 1

$(\sqrt{5}\sqrt{2} + \sqrt{2})(\sqrt{5}\sqrt{3} - \sqrt{2})$

or

$\sqrt{5}\sqrt{5}\sqrt{2}\sqrt{3} + \sqrt{5}\sqrt{2}\sqrt{3} - \sqrt{2}\sqrt{5}\sqrt{3} - \sqrt{2}\sqrt{3}$

or better …

Allow one error (sign or term) in the expansion

M1

Eliminating the two 'middle' terms

These must be the correct two middle terms

M1

$\sqrt{5}\sqrt{5}\sqrt{2}\sqrt{3}$ simplified to $5\sqrt{6}$

M1

$4\sqrt{6}$

A1

Alternative method 2

$(\sqrt{5}\sqrt{2} + \sqrt{2})(\sqrt{5}\sqrt{3} - \sqrt{3})$

M1

$\sqrt{2}\sqrt{3}(\sqrt{5} + 1)(\sqrt{5} - 1)$

M1

$\sqrt{2}\sqrt{3} \times (5 - 1)$

M1

$4\sqrt{6}$

A1

[4]